Fill in the following identities. **SCORE: ____ / 14 PTS**

[a] SUM OF ANGLES IDENTITY:

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

[b] DIFFERENCE OF ANGLES IDENTITY:

$$\sin(x-y) = \sin x \cos y - \cos x \sin y$$

[c] HALF ANGLE IDENTITY:

$$\sin\tfrac{1}{2}x = \pm\sqrt{\frac{1-\cos x}{2}}$$

[d] POWER REDUCING IDENTITY:

$$\cos^2 x = \frac{1+\cos 2x}{2}$$

[e] PYTHAGOREAN IDENTITY:

$$\tan^2 x = \sec^2 x - 1$$

[f] NEGATIVE ANGLE IDENTITY:

$$\cos(-x) = \cos x$$

[g] DOUBLE ANGLE IDENTITY: WRITE ALL 3 VERSIONS

$$\cos 2x = \cos^2 x - \sin^2 x,\ 2\cos^2 x - 1,\ 1 - 2\sin^2 x$$

If $\cos t = \frac{2}{3}$ and $\frac{3\pi}{2} < t < 2\pi$, find the values of the following expressions. **SCORE: ____ / 30 PTS**

Write each final answer as a single fraction in simplest form, including rationalizing the denominator.

[a] $\tan\frac{1}{2}t$

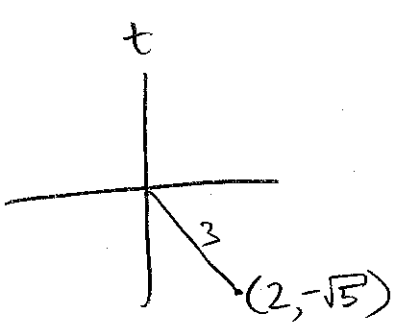
$$= \csc t - \cot t$$
$$= \frac{3}{-\sqrt{5}} - \frac{2}{-\sqrt{5}}$$
$$= -\frac{1}{\sqrt{5}}$$
$$= -\frac{\sqrt{5}}{5}$$

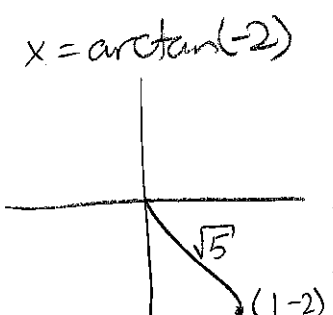
[b] $\sin 2t$

$$= 2\sin t \cos t$$
$$= 2\left(-\frac{\sqrt{5}}{3}\right)\left(\frac{2}{3}\right)$$
$$= -\frac{4\sqrt{5}}{9}$$

[c] $\cos(\underbrace{\arctan(-2)}_{x} - t)$

$$= \cos x \cos t + \sin x \sin t$$
$$= \frac{2}{3}\cdot\frac{1}{\sqrt{5}} + \frac{-\sqrt{5}}{3}\cdot\frac{-2}{\sqrt{5}}$$
$$= \frac{2+2\sqrt{5}}{3\sqrt{5}} = \frac{2\sqrt{5}+10}{15}$$





Solve the equation $4(1-\cos\frac{1}{3}x)=5-2\cos\frac{1}{3}x$.

SCORE: ____ / 14 PTS

$$4-4\cos\tfrac{1}{3}x=5-2\cos\tfrac{1}{3}x$$
$$-2\cos\tfrac{1}{3}x=1$$
$$\cos\tfrac{1}{3}x=-\tfrac{1}{2}$$
$$\tfrac{1}{3}x=\tfrac{2\pi}{3}+2n\pi \text{ or } \tfrac{4\pi}{3}+2n\pi$$
$$x=2\pi+6n\pi \text{ or } 4\pi+6n\pi$$

Prove the identity $\dfrac{\csc^2 x-\sec^2 x}{\cot x-\tan x}=\csc x\sec x$.

SCORE: ____ / 14 PTS

$$\hookrightarrow=\frac{(1+\cot^2x)-(1+\tan^2x)}{\cot x-\tan x}$$
$$=\frac{\cot^2x-\tan^2x}{\cot x-\tan x}$$
$$=\frac{(\cot x+\tan x)(\cot x-\tan x)}{\cot x-\tan x}$$
$$=\cot x+\tan x$$
$$=\frac{\cos x}{\sin x}+\frac{\sin x}{\cos x}$$
$$=\frac{\cos^2x+\sin^2x}{\sin x\cos x}$$
$$=\frac{1}{\sin x\cos x}$$
$$=\csc x\sec x$$

Rewrite $\cos^2 x\sin^2 x$ using only the first powers of cosine (and constants and the 4 basic arithmetic operations). **SCORE: ____ / 14 PTS**
Simplify your final answer, which must **NOT** be in factored form, and must **NOT** involve any other trigonometric functions.

$$\frac{1+\cos 2x}{2}\cdot\frac{1-\cos 2x}{2}$$
$$=\frac{1-\cos^2 2x}{4}$$
$$=\frac{1-\left(\frac{1+\cos 4x}{2}\right)}{4}\cdot\frac{2}{2}$$
$$=\frac{2-(1+\cos 4x)}{8}=\frac{1-\cos 4x}{8}$$